RAPID COMMUNICATIONS

PHYSICAL REVIEW C VOLUME 42, NUMBER 6 DECEMBER 1990

y scaling as a probe of nuclear light-cone dynamics

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(Received 7 November 1989)

The y scaling exhibited in quasielastic electron scattering on nuclei is shown to occur in the same kinematic limit as x scaling in deep-inelastic lepton-nucleon scattering. Using the impulse approximation in a relativistic model, we demonstrate that the scaling function $F(y)$ can be interpreted as the nucleon light-cone momentum distribution and the scaling variable y is related to the light-cone momentum t^+ of the nucleon. We also derive the convolution formula for deep-inelastic lepton-nucleus scattering and show that the same $F(y)$ can be extracted from the experimental structure functions of the nucleon and nuclei.

Scaling phenomena have been studied extensively in scattering processes for the past two decades. The attractive aspect of scaling is that its occurrence often (if not always) suggests that a simple reaction mechanism, such as the impulse approximation, dominates the process, and therefore allows one to extract structural information in a model independent way. The best-known example of scaling is in deep-inelastic electron-nucleon scattering, where the structure function F_2, which is in general a function of the four-momentum transfer squared (Q^2) and energy transfer (ν), exhibits scaling in the Bjorken limit[1] ($Q^2 \rightarrow \infty, \nu \rightarrow \infty, Q^2/\nu =$fixed); i.e., it becomes only a function of $x = Q^2/2M\nu$. In this limit the scaling function $F_2(x)/x$ represents the parton light-cone momentum distribution in the nucleon ground state.

A less well-studied example is quasielastic electron scattering from a nucleus at large momentum transfer. In this case, the "reduced" total cross section was predicted,[2] and subsequently observed,[3] to scale according to a variable $y(q,\nu)$. However, the definition of both the scaling function $F(y)$ and the scaling variable has been the subject of considerable discussion (see Ref. 4 for a summary). The original theoretical study by West obtained y scaling with the assumptions of on-shell initial nucleons and nonrelativistic kinematics. The experimental reduced cross section does not scale according to the West scaling variable, but does so with a relativistic variable,

$$y = -q_3 + [(\nu+\epsilon)^2 + 2M(\nu+\epsilon)]^{1/2}, \tag{1}$$

where q_3 is the three-momentum transfer, M is the nucleon mass, and the initial nucleon is assumed to have an average binding ϵ. The scaling variable y has been interpreted as the projection of the initial nucleon three momentum in the direction of the virtual photon and the scaling function $F(y)$ has been simply related to the nucleon momentum distribution. This interpretation has recently been questioned by studies of more general impulse approximation models for few and many-body systems.[5-7] In particular, the short-range nuclear correlations modify the nucleon missing energy distribution and a model-independent extraction of the nucleon momentum density from $F(y)$ is not possible.

Here we attempt to clarify the meaning of the y-scaling variable and the scaling function by comparison with Bjorken x scaling. From the definition of y, we show that the y-scaling limit is the same as the Bjorken limit and that in this limit both scaling variables can be related. Then with some general assumptions, we derive the relativistic impulse approximation (RIA) for the two reduced nuclear structure functions of quasielastic scattering, both of which are shown to scale in the Bjorken limit. As a consequence, one can define a "reduced" cross section which exhibits y scaling. (To author's knowledge such a derivation of y scaling in *relativistic* impulse approximation has never been done before). We arrange the kinematic factors so that the scaling function $F(y)$ has a meaning similar to the parton distribution in the nucleon. The scaling variable y is shown to relate to the nucleon light-cone momentum t^+. By extending the quasielastic RIA structure function to deep-inelastic lepton-nucleus scattering, we obtain a convolution formula for the deep-inelastic structure functions.[8] This formula depends on the same scaling function $F(y)$, and therefore one can obtain information on the nucleon light-cone density by combining the deep-inelastic scattering data on nucleons and nuclei.

First we show that the y-scaling limit is the same as the Bjorken limit from the definition of y at finite Q^2,[6]

$$y = -q_3 + [(\nu+E_s)^2 + 2M(\nu+E_s)]^{1/2}, \tag{2}$$

where E_s is the single-nucleon separation energy of the nucleus. The nuclear recoil energy is included in the definition of the associated spectral function. For fixed y, one can solve for ν in the y-scaling limit ($q_3 \rightarrow \infty$),

$$\nu = q_3 + y - M - E_s. \tag{3}$$

Substituting this into the definition of the Bjorken variable x, we have

$$x=\frac{Q^2}{2M\nu}=1+\frac{E_s-y}{M}=\frac{M_A}{M}\tilde{y}_A\,, \tag{4}$$

or

$$y=M(1-x)+E_s=M-M_A\tilde{y}_A+E_s\,, \tag{5}$$

where M_A is the mass of the nucleus. For $x=1$, y is near zero (the quasielastic peak), for $x=0$, y is $M+E_s$ (the kinematic cutoff), and for $x=2$, y is $-M$ (the elastic deuteron peak). For different definitions of y, the relationship between x and y can change in the scaling limit, but y scaling still requires Q^2/ν to be finite as $Q^2\rightarrow\infty$. In the subsequent discussion, we will actually take the Bjorken limit when we say the y-scaling limit.

Now we discuss the relativistic impulse approximation for quasielastic scattering and show that the reduced cross section scales in the scaling limit. For simplicity, we will consider quantum-field-theory models with the nucleon as the fundamental degree of freedom. The model Lagrangians presumably are derivable from quantum chromodynamics (QCD), but their detailed form will not be important for our discussion. With the nucleon as the fundamental degree of freedom, the electromagnetic current (neglecting the possible meson contributions) is

$$J_\mu(r)=\bar{\psi}(r)\Gamma_\mu\psi(r)\,, \tag{6}$$

where Γ_μ has contributions from both the Dirac and Pauli terms. The nuclear tensor, from which the total inclusive cross section can be derived, is given by

$$W_{\mu\nu}(q)=\frac{1}{4\pi}\int e^{iqr}d^4r\langle 0|[J_\mu(r),J_\nu(0)]|0\rangle\,. \tag{7}$$

The commutator is difficult to evaluate in general because it is not at equal time and one needs the Hamiltonian of the system to translate the current between different times.

Under the impulse approximation, which is defined to be no interactions between the struck nucleon propagating from 0 to r_μ and the rest of the system, the commutator can be calculated using the free-field commutation relations. To simplify the discussion, we will assume the antinucleon is unimportant and neglect antinucleon operators in the Fourier expansion of the fields (there is not clear experimental evidence that the antinucleon degrees of freedom are important in nuclear physics). Then the nuclear tensor becomes

$$W_{\mu\nu}=\frac{1}{4\pi}\int d^4r\,e^{iqr}\int\frac{d\mathbf{k}}{2E_{\mathbf{k}}(2\pi)^2}\sum_\lambda[e^{-ikr}(\Gamma_\mu u_{\mathbf{k}\lambda}\bar{u}_{\mathbf{k}\lambda}\Gamma_\nu)_{\alpha\beta}\langle 0|\bar{\psi}_\alpha(r)\psi_\beta(0)|0\rangle+e^{ikr}(\Gamma_\nu u_{\mathbf{k}\lambda}\bar{u}_{\mathbf{k}\lambda}\Gamma_\mu)_{\alpha\beta}\langle 0|\bar{\psi}_\alpha(0)\psi_\beta(r)|0\rangle]\,, \tag{8}$$

where $E_{\mathbf{k}}=(M^2+\mathbf{k}^2)^{1/2}$ and the u's are Dirac spinors. The second term in the bracket arises from the Pauli-principle correction[9] and can be neglected for large momentum transfers. Using the Heisenberg equation of motion to translate $\bar{\psi}_\alpha(r)$ to $\bar{\psi}_\alpha(0)$ and integrating over r_μ, we have

$$W_{\mu\nu}=\frac{(2\pi)^4}{4\pi}\int d^4t\sum_\lambda\int\frac{d\mathbf{k}}{2E_{\mathbf{k}}(2\pi)^3}\delta(t+q-k)(\Gamma_\mu u_{\mathbf{k}\lambda}\bar{u}_{\mathbf{k}\lambda}\Gamma_\nu)_{\alpha\beta}\langle 0|\bar{\psi}_\alpha(0)\delta^4(P^0-\hat{P}-t)\psi_\beta(0)|0\rangle\,, \tag{9}$$

where we have introduced the covariant four-momentum $t=(t_0,\mathbf{t})$ of the struck particle. $\hat{P}$ is the four-momentum operator of the system and P^0 is its ground-state eigenvalue. Expanding the field operator in terms of the free-space momentum eigenstates, we get

$$W_{\mu\nu}=\frac{1}{2}\sum_{\lambda,\lambda',\lambda''}\int d^4t\frac{2M_A}{2E_{\mathbf{t+q}}2E_{\mathbf{t}}}\delta(t_0+\nu-E_{\mathbf{t+q}})\mathrm{Tr}(\Gamma_\mu u_{t+q,\lambda}\bar{u}_{t+q,\lambda}\Gamma_\nu u_{\bar{t},\lambda'}\bar{u}_{\bar{t},\lambda''})S_{\lambda',\lambda''}(t)\,, \tag{10}$$

where $\bar{t}=(E_{\mathbf{t}},\mathbf{t})$ is the on-shell energy momentum and is not covariant in the presence of interactions and $\{S_{\lambda',\lambda''}(t)\}=\mathbf{S}(t)$ is the spectral function matrix,

$$\mathbf{S}(t)=\frac{1}{(2\pi)^6 2E_{\mathbf{t}}}\sum_n\langle 0|a^\dagger_{\mathbf{t},\lambda''}|n\rangle\langle n|a_{\mathbf{t},\lambda'}|0\rangle\times\delta(M_A-t_0-P_0^n)\,. \tag{11}$$

Approximating $S_{\lambda',\lambda''}(t)$ by a spin average,

$$S_{\lambda',\lambda''}(t)\approx\delta_{\lambda',\lambda''}S(t)\equiv\tfrac{1}{2}\delta_{\lambda',\lambda''}\mathrm{Tr}\mathbf{S}(t)\,,$$

we obtain,

$$W_{\mu\nu}=\int d^4t\frac{M_A}{E_{\mathbf{t}}}w_{\mu\nu}S(t)\,, \tag{12}$$

where $w_{\mu\nu}$ is the single-nucleon tensor,

$$w_{\mu\nu}=\delta(t_0+\nu-E_{\mathbf{t+q}})h_{\mu\nu}\,, \tag{13}$$

$$h_{\mu\nu}2E_{\mathbf{t+q}}=\frac{1}{4}\sum_{\lambda,\lambda'}\mathrm{Tr}(\Gamma_\mu u_{t+q,\lambda}\bar{u}_{t+q,\lambda}\Gamma_\nu u_{\bar{t},\lambda'}\bar{u}_{\bar{t},\lambda'})\,. \tag{14}$$

It should be emphasized that $w_{\mu\nu}$ here is not a Lorentz tensor, neither is $S(t)$ a scalar, due to the presence of $\bar{t}$. However, the nuclear tensor $W_{\mu\nu}$ is a Lorentz tensor, as can be seen from Eq. (9). Furthermore, the current conservation is not obeyed at the single nucleon level. To our view, this is natural for a bounded nucleon. The current conservation should only be imposed on the total response tensor, $W_{\mu\nu}$. The nucleon tensor can be evaluated easily from Eq. (14) and gives,

$$h_{\mu\nu}2E_{t+q}=h_1\left(-g_{\mu\nu}+\frac{\bar{q}_\mu\bar{q}_\nu}{\bar{q}^2}\right)+\frac{h_2}{M^2}\left(\bar{t}_\mu-\frac{(\bar{t}\cdot\bar{q})\bar{q}_\mu}{\bar{q}^2}\right)\left(\bar{t}_\nu-\frac{(\bar{t}\cdot\bar{q})\bar{q}_\nu}{\bar{q}^2}\right), \tag{15}$$

where

$$h_1=\frac{\bar{Q}^2}{2}G_M^2(Q^2), \tag{16}$$

$$h_2=2M^2\frac{G_E^2+\bar{Q}^2/4M^2G_M^2}{1+\bar{Q}^2/4M^2}, \tag{17}$$

and $\bar{q}_\mu=[\bar{\nu}=\nu-(E_t-t_0),\mathbf{q}]$, $\bar{Q}^2=-\bar{q}^2$. We will also need $\tilde{h}_i=h_i/2E_{t+q}$ in later discussion.

The quasielastic structure functions for the nucleus can be extracted from the nuclear tensor,

$$W_1=-\tfrac{1}{2}\int d^4t\frac{M_A}{E_t}\left\{w_1\left(-3+\frac{q^2}{\bar{q}^2}\right)+\frac{w_2}{M^2}\left[\left(\bar{t}^2-\frac{(\bar{q}\cdot\bar{t})^2}{\bar{q}^2}\right)+\frac{q^2}{\mathbf{q}^2}\left(E_t-\frac{(\bar{q}\cdot\bar{t})\bar{\nu}}{\bar{q}^2}\right)^2\right]\right\}S(t) \tag{18}$$

and

$$W_2=\tfrac{1}{2}\int d^4t\frac{M_A}{E_t}\frac{q^2}{\mathbf{q}^2}\left\{w_1\left(-3+3\frac{q^2}{\bar{q}^2}\right)+\frac{w_2}{M^2}\left[\left(\bar{t}^2-\frac{(\bar{q}\cdot\bar{t})^2}{\bar{q}^2}\right)+3\frac{q^2}{\mathbf{q}^2}\left(E_t-\frac{(\bar{q}\cdot\bar{t})\bar{\nu}}{\bar{q}^2}\right)^2\right]\right\}S(t). \tag{19}$$

These two equations represent the main result of the relativistic impulse approximation with the binding correction, $\Delta t=E_t-t_0$, appearing both in the kinematic coefficients and in the spectral function. If we set $\Delta t=0$, we are back to the usual Fermi gas model.[10] In the scaling limit, W_1 and W_2 become

$$W_1=\int d^4t\frac{M_A}{E_t}\left(\frac{M_A\tilde{y}_A+\frac{3}{2}\Delta t}{M_A\tilde{y}_A+\Delta t}w_1-\frac{\nu\Delta t(E_t+t_3)^2}{4(M_A\tilde{y}_A+\Delta t)^2}\frac{w_2}{M^2}\right)S(t), \tag{20}$$

and

$$W_2=\int d^4t\frac{M_A}{E_t}\left(\frac{M_A\tilde{y}_A(2M_A\tilde{y}_A-\Delta t)(E_t+t_3)^2}{2(M_A\tilde{y}_A+\Delta t)^2}\frac{w_2}{M^2}+\frac{3M_A\tilde{y}_A\Delta t}{\nu(M_A\tilde{y}_A+\Delta t)}w_1\right)S(t), \tag{21}$$

where the elastic nucleon structure functions are

$$w_1=\tfrac{1}{2}(M_A\tilde{y}_A+\Delta t)G_M^2(Q^2)\delta(\nu-q+t_0+t_3), \tag{22}$$

$$w_2=M^2G_M^2(Q^2)/\nu\delta(\nu-q+t_0+t_3). \tag{23}$$

According to Eq. (4), $\tilde{y}_A=(q-\nu)/M_A$, we write the δ-function argument as $\delta(\tilde{y}_AM_A-\sqrt{2}t^+)$. Substituting w_1 and w_2 into Eqs. (20) and (21) we see that $F_1=W_1/G_M^2(Q^2)$ and $F_2=W_2\nu/M_AG_M^2(Q^2)$ are functions of $\tilde{y}_A$, or $y=M-M_A\tilde{y}_A+E_s$, only,

$$F_1(y)=\frac{M_A}{2}\int d^4t\left(1+\frac{t_3}{E_t}\right)\delta(\tilde{y}_AM_A-\sqrt{2}t^+)S(t), \tag{24}$$

$$F_2(y)=\tilde{y}_AM_A\int d^4t\left(1+\frac{t_3}{E_t}\right)\delta(\tilde{y}_AM_A-\sqrt{2}t^+)S(t), \tag{25}$$

where we have eliminated $\tilde{y}_A$ using the δ-function condition $\tilde{y}_A=t^+/P^+$ with $P^+=M_A/\sqrt{2}$. A few important conclusions follow from Eqs. (24) and (25) immediately. First, the binding effects in the kinematic coefficients cancel out completely and we are left with a factor, $1+t_3/E_t$, which is usually called the flux factor. Second, the δ function indicates that y is related to the nucleon light-cone momentum t^+, not just t_3. Finally, the Callan-Gross relation $F_2=2\tilde{y}_AF_1$ holds in the impulse approximation.

It is then easy to show that the reduced quasielastic cross-section scales. Substituting W_1 and W_2 into the Born-approximation cross-section formula,[1] and dividing both sides by the limiting total elastic cross sections for N neutrons and Z protons, we have

$$F(y)=MM_A\frac{d^2\sigma/d\nu d\epsilon}{\sigma_M[H_2\tilde{y}+2H_1\tilde{y}\tan^2(\theta/2)]}=M_A\int d^4t\left(1+\frac{t_3}{E_t}\right)\delta(\tilde{y}_AM_A-\sqrt{2}t^+)S(t), \tag{26}$$

where $H_i=N\tilde{h}_i^N+Z\tilde{h}_i^P$ $(i=1,2)$ and $\tilde{y}=M_A/M\tilde{y}_A$; $\tilde{h}_i$ was defined after Eq. (17). The reduced cross section $F(y)$ is just $2F_1(y)=F_2(y)/\tilde{y}_A$ defined in Eqs. (24) and (25), which can be interpreted as the nucleon distribution function on the light cone by a similar identification to x scaling. It differs from the original definition of $F(y)$ by the flux factor,[5,11] which in our case arises from the consistent treatment of the relativistic kinematics. In Ref. 11, the experimental scaling function was obtained with this flux factor divided out from the measured cross section. To maintain the physical meaning of the scaling function, the flux factor must be included in its definition. This same flux factor was particularly emphasized in the literature (see, e.g., Ref. 12) in explaining the European Muon Collaboration (EMC) data, although it was interpreted differently.

Since the Bjorken variable x has an upper limit, M_A/M, there must be a lower limit on y,

$$y > M - M_A + E_s = -M_{A-1}\,, \tag{27}$$

where M_{A-1} is the ground-state mass of the $A-1$ system. On the other hand, there are no physical upper limits on y. The allowed range for y is then $(-M_{A-1},\infty)$. However, in quasielastic electron-nucleus scattering the spacelike photon gives a lower limit $x=0$ and thus y can only be accessed up to $M+E_s$. The presence of limits on y reinforces our identification of y with t^+ rather than with a longitudinal nucleon momentum which, in principle, can vary from $-\infty$ to $+\infty$.

Note that the scaling function $F(y)$ is not symmetric around $y=0$ as can be seen from the theoretical expression (26). The sum rule on $F(y)$ can be derived easily from the same equation,

$$\int_{-M_{A-1}}^{\infty} F(y)\frac{dy}{M_A} = 1\,. \tag{28}$$

In Ref. 11, the sum rule is calculated from $y=-M_{A-1}$ to 0 because the $y>0$ part of the data is contaminated by the nucleon in inelastic contribution. The result for a series of nuclei shows that the numbers in general less than half, which indicates that the $y>0$ part has a larger contribution to the sum rule, consistent with the impulse approximation.

Having made a close analogy between y scaling in quasielastic electron-nucleus and deep-inelastic x scaling, we now turn to the deep-inelastic lepton-nucleus scattering where both phenomena should occur. There are many discussions in the literature about this process in connection with the EMC effect.[12] Here we use the impulse approximation formula (20) for W_2 to derive the convolution model.[8] From the result, we conclude that the same $F(y)$ measured in quasielastic electron scattering occurs also in the deep-inelastic process.

Going back to the formula (20) and disregarding the binding effects, we have

$$W_2 = \int d^4t \frac{M_A}{E_t}\frac{(E_t+t_3)^2}{M^2} w_2 S(t)\,. \tag{29}$$

We generalize this formula to inelastic scattering from the nucleus by considering w_2—the inelastic structure function of the nucleon. Using the definition of $F_2^N(x)$ for the nucleon structure function, we get

$$F_2^A(x_A) = \int d^4t \left(1+\frac{t_3}{E_t}\right) S(t) F_2^N[x_A M_A/(t_0+t_3)]\,, \tag{30}$$

where $x_A = M/M_A x$. Inserting a δ function $\delta[\tilde{y}_A M_A - (t_0+t_3)]$ and using the definition of $F(\tilde{y}_A)$, Eq. (26), we have,

$$F_2^A(x_A) = \int_{x_A}^{1} d\tilde{y}_A F(\tilde{y}_A) F_2^N(x_A/\tilde{y}_A)\,. \tag{31}$$

Note the relationship between x_A and $\tilde{y}_A$ in Eq. (4) is invalid for inelastic scattering. Instead, $\tilde{y}_A > x_A$. If the *nucleon* structure function on the right-hand side and the *nuclear* structure function on the left-hand side are measured over the whole range of x values, we can solve the integral equation to obtain $F(y)$. For instance, we could take moments of both sides of the equation and then the moments of $F(y)$ are just equal to ratios of the nuclear and nucleon structure functions. Comparisons of $F(y)$ obtained from this procedure with the same quantity in quasielastic scattering could shed some light on our understanding of both processes. In this respect, future $x>1$ experiments would be very useful in exploring the properties of $F(y)$ at large momentum transfers. We will discuss this aspect in more detail in a forthcoming publication.[13]

So far, we have not addressed final-state interactions. We implicitly assumed the final-state effects would vanish when the momentum transfer is large enough and the impulse approximation cross section finally dominates the process. However, the present experimental data do not rule out the possibility that the observed scaling is due to final-state interactions. In fact even asymptotically free QCD has a logarithmic Q^2 correction on the parton densities at all Q^2. This should serve as a strong warning about the validity of the impulse approximation at large Q^2. However, one can still define a nucleon light-cone density and study its Q^2 evolution, which could place useful constraints on the dynamical structure of nuclear models.

This work was supported in part by funds provided by the U.S. Department of Energy under Contract No. DE-AC02-76ER03069, the National Science Foundation under Grants No. PHY88-17296 and No. PHY86-04197, and by the Sloan Research Foundation (B.W.F.).

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